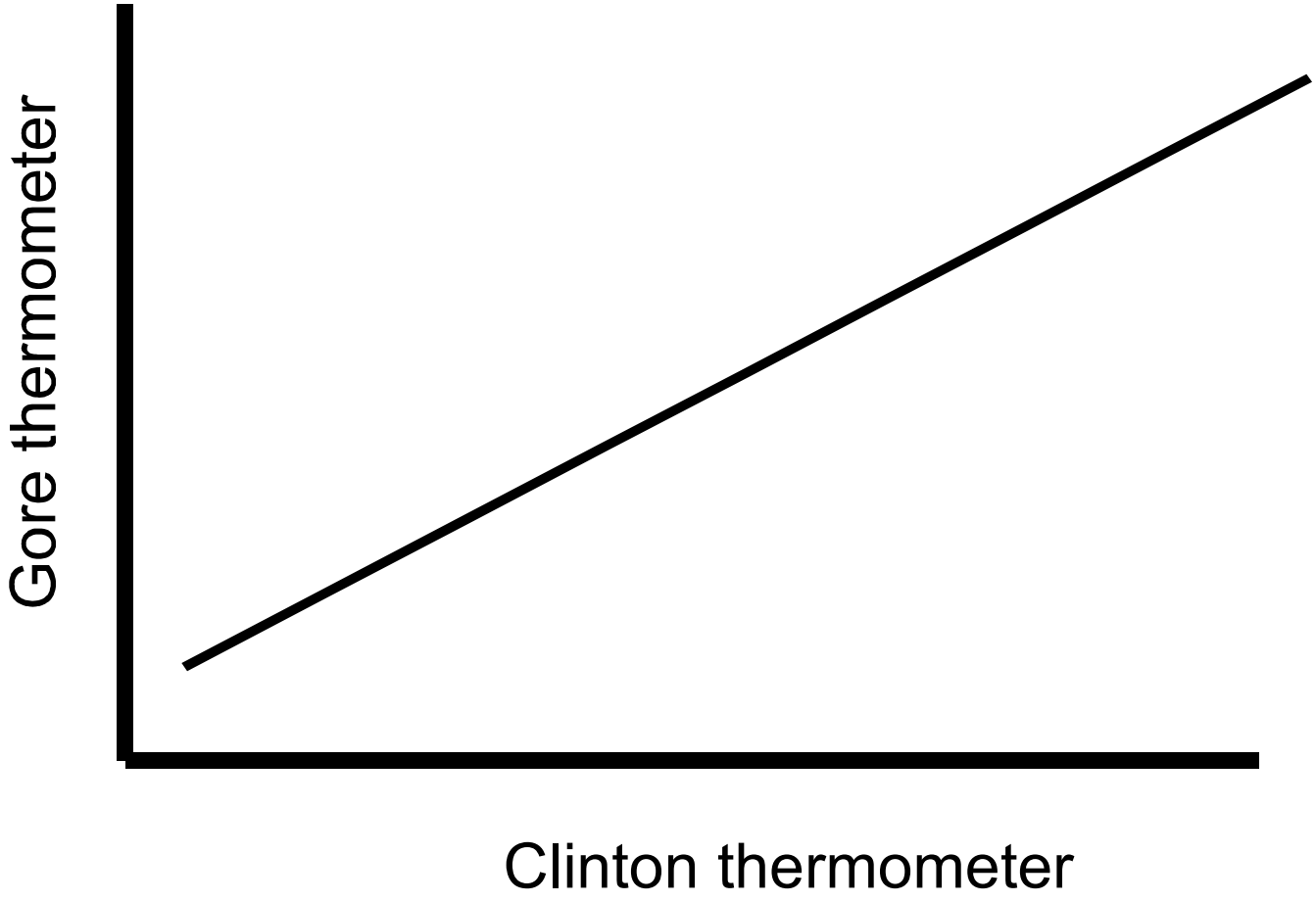
# Addressing Alternative Explanations: Multiple Regression

17.871

# Did Clinton hurt Gore example

- Did Clinton hurt Gore in the 2000 election?
  - Treatment is not liking Bill Clinton
- How would you test this?

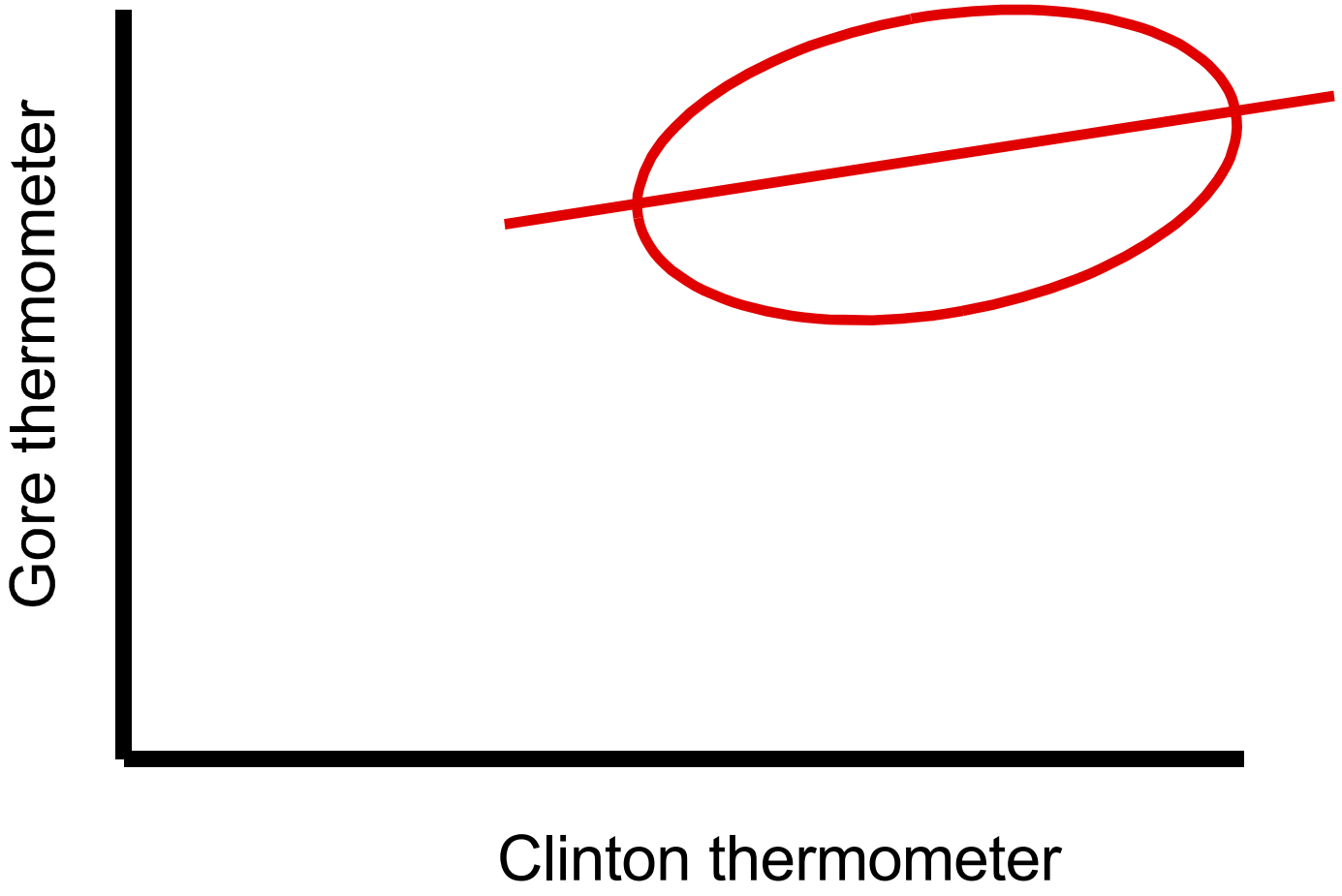
# Bivariate regression of Gore thermometer on Clinton thermometer



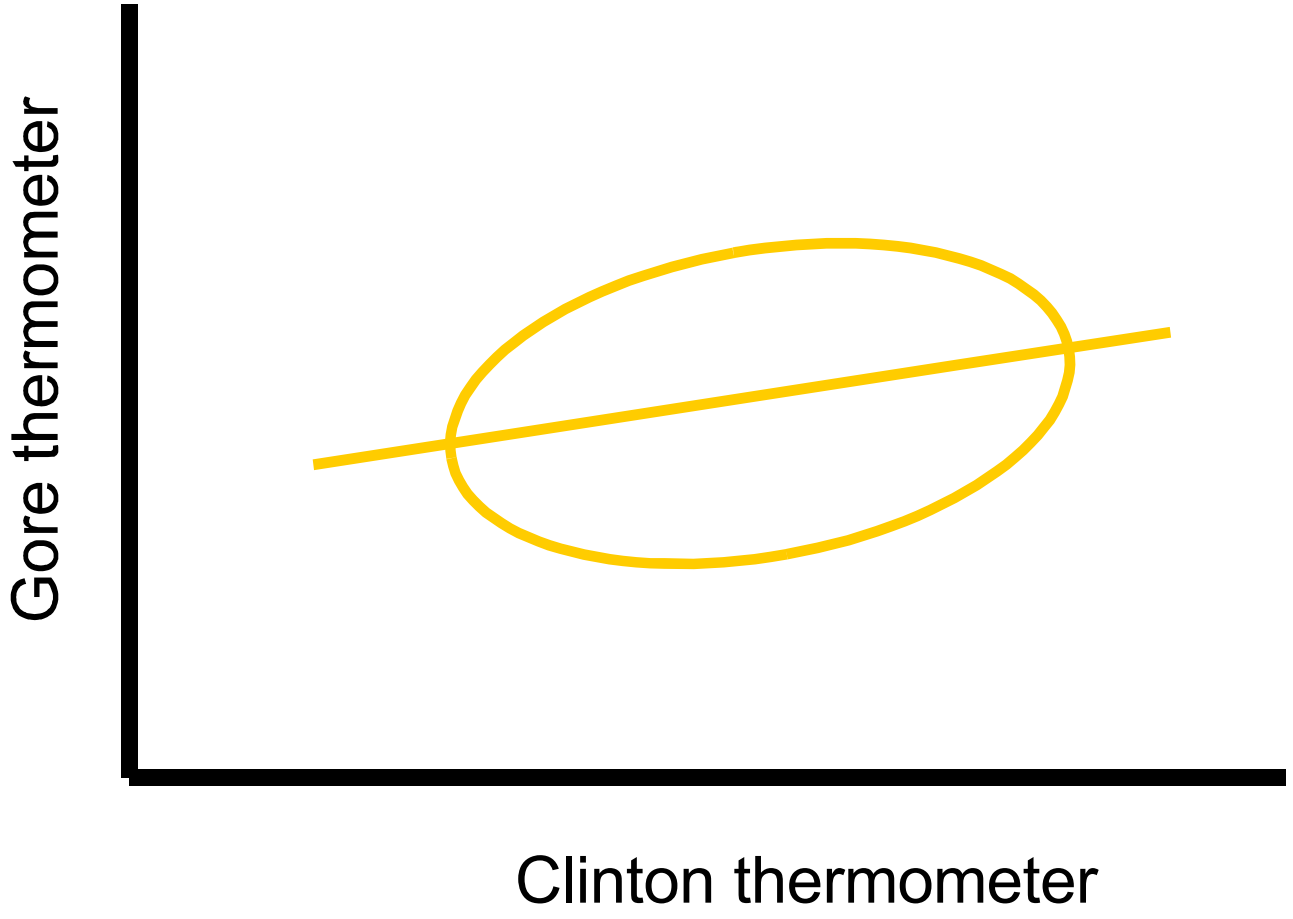
# Did Clinton hurt Gore example

- What alternative explanations would you need to address?
- Nonrandom selection into the treatment group (disliking Clinton) from many sources
- Let's address one source: party identification
- How could we do this?
  - Matching: compare Democrats who like or don't like Clinton; do the same for Republicans and independents
  - Multivariate regression: control for partisanship statistically

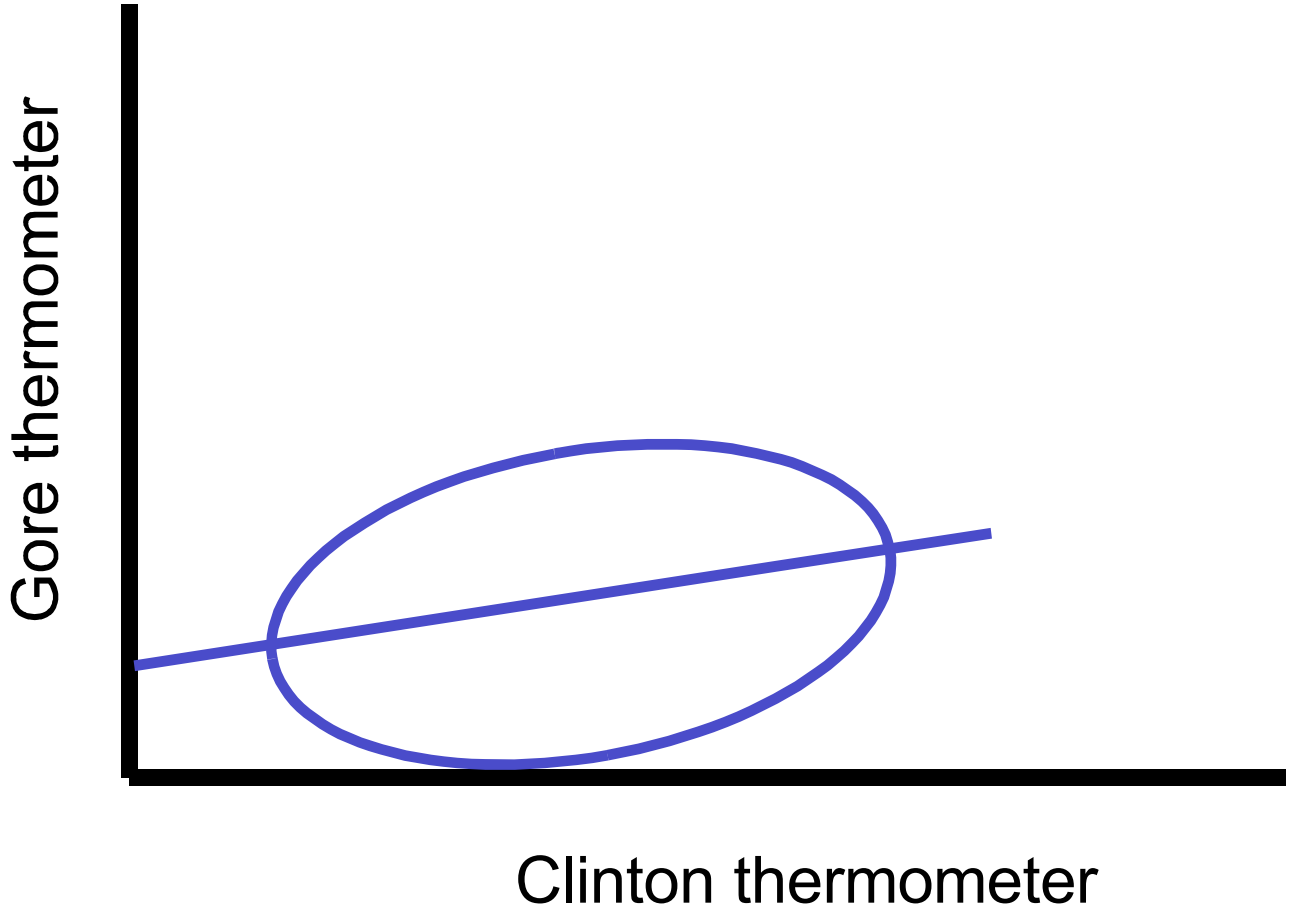
# Democratic picture



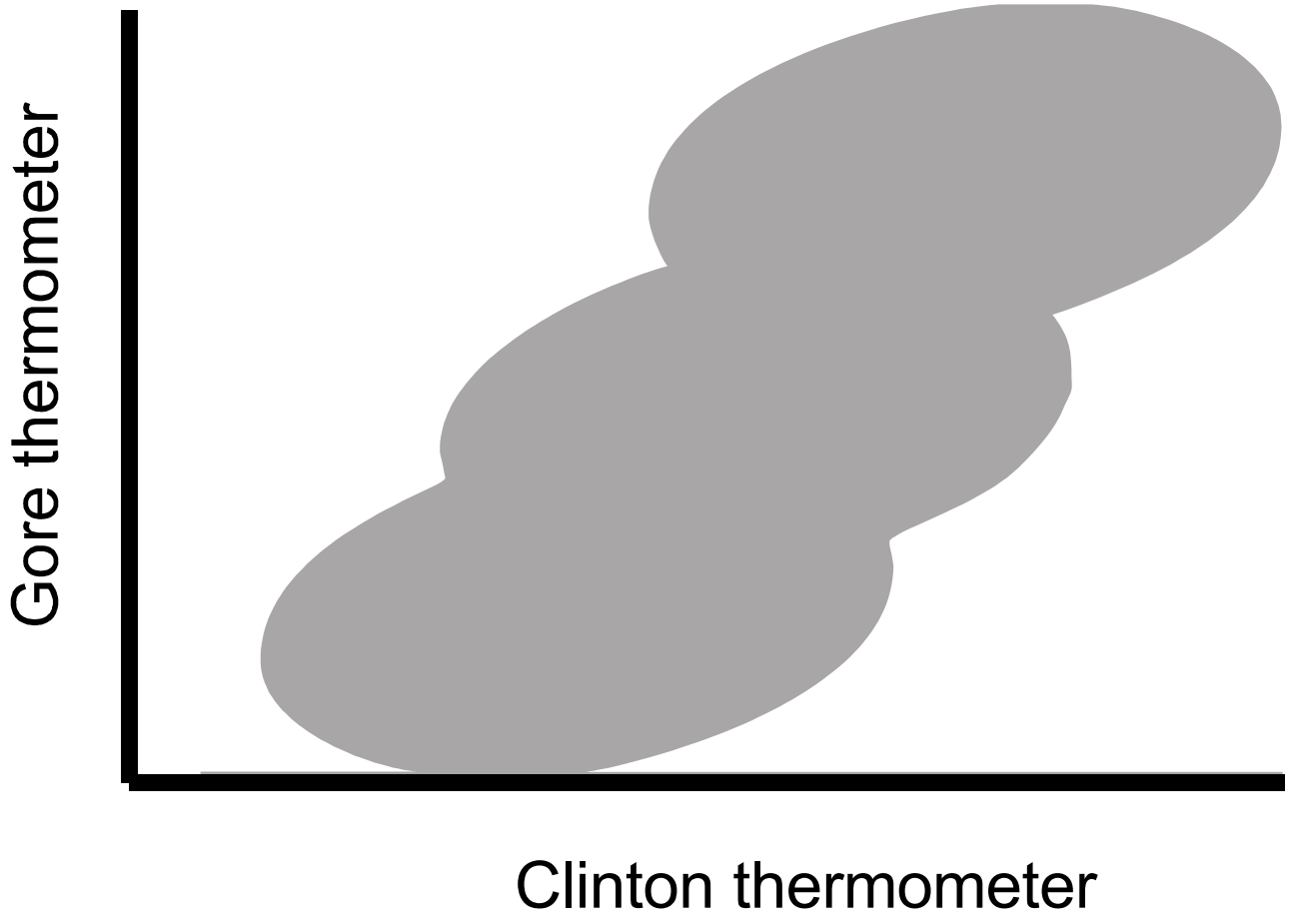
# Independent picture



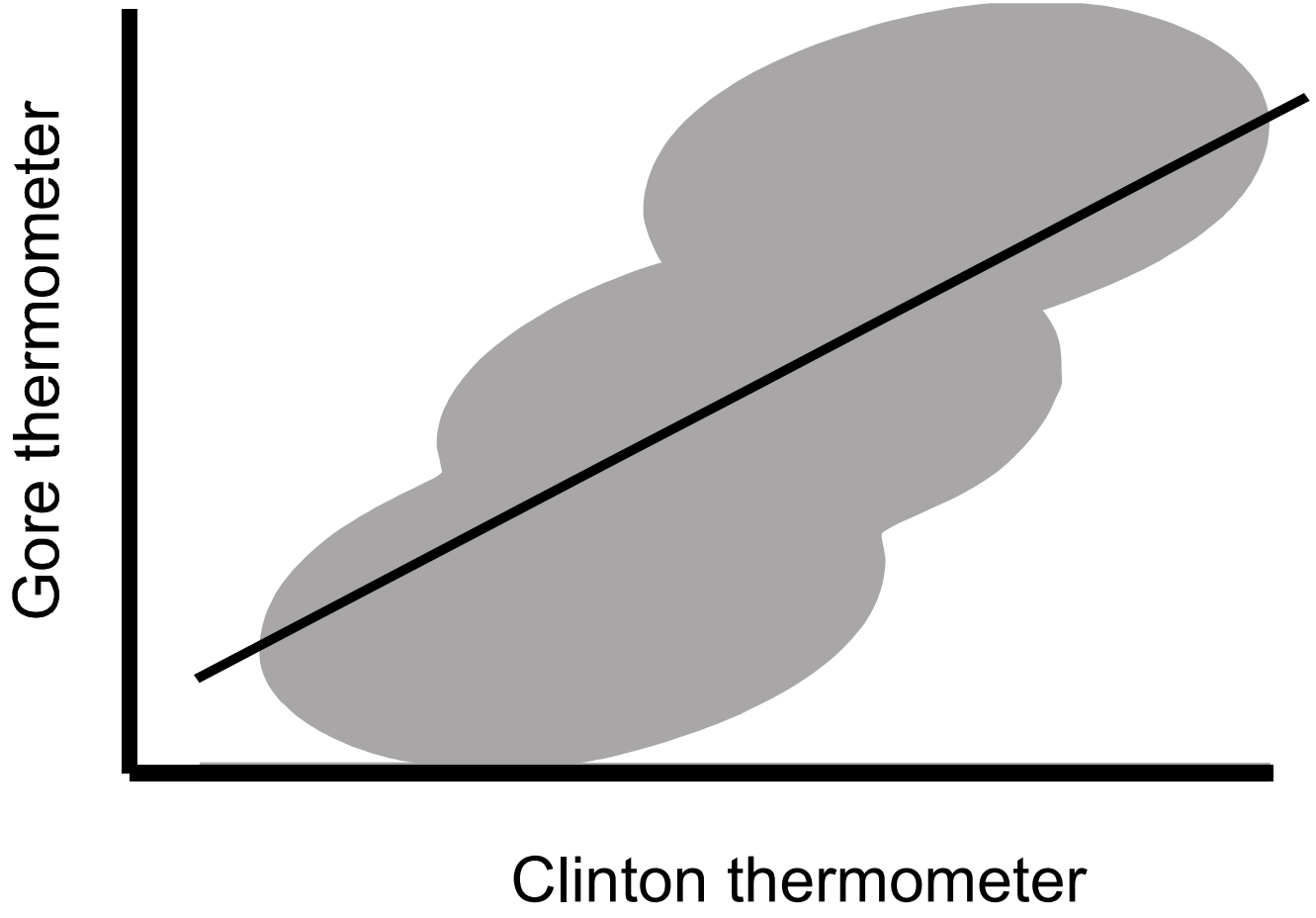
# Republican picture



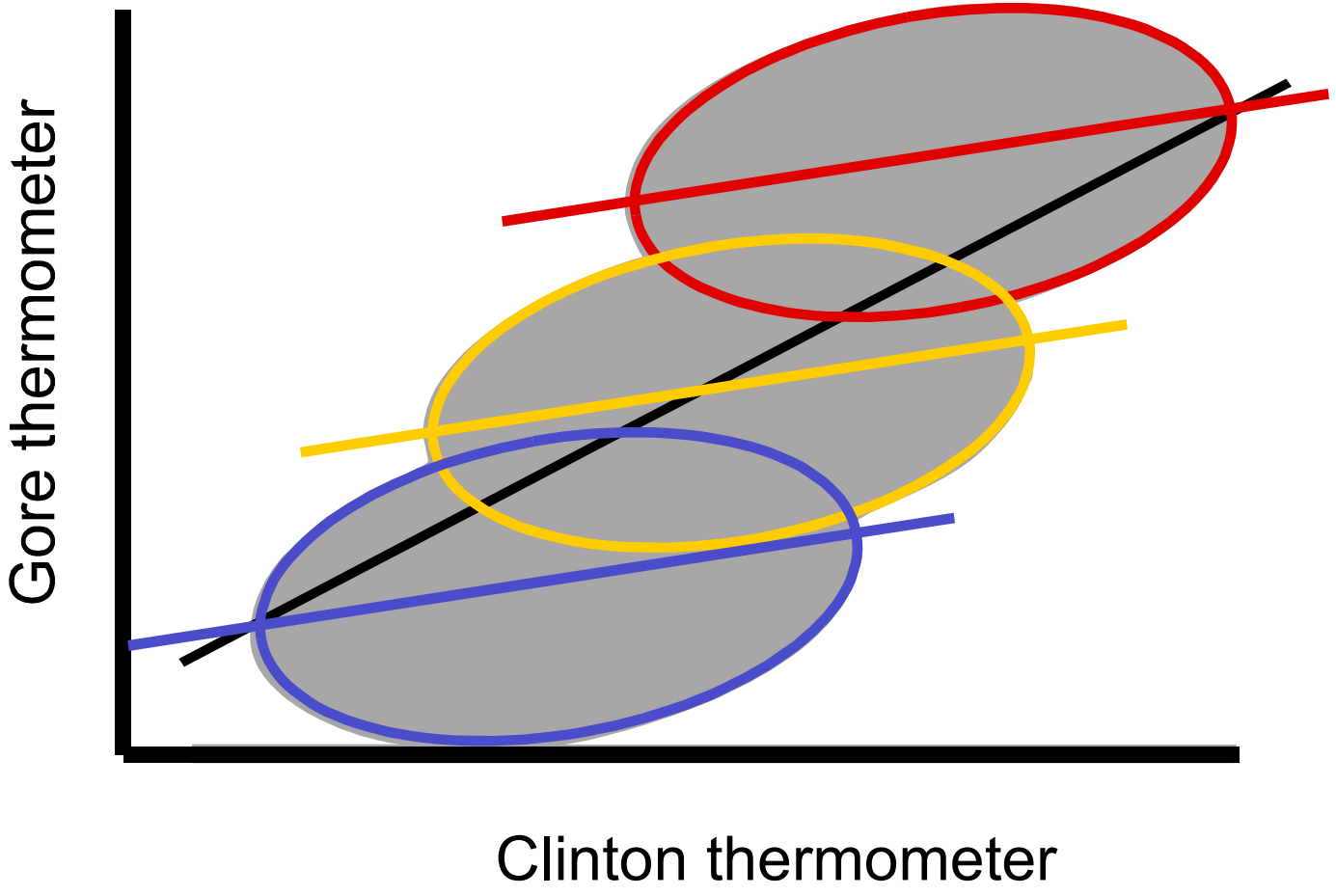
# Combined data picture



# Combined data picture with regression: bias!

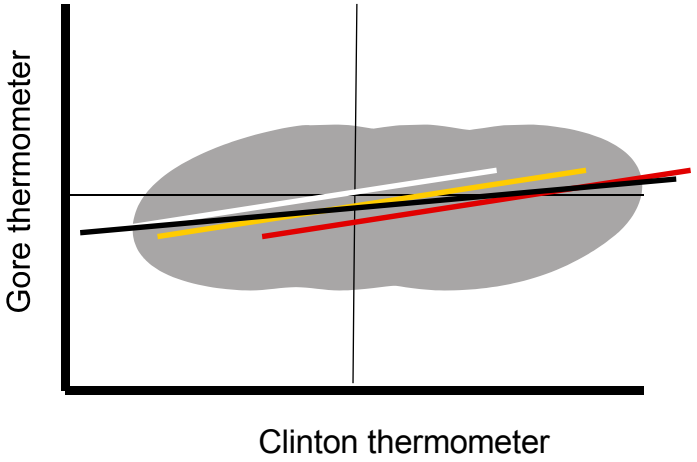


# Combined data picture with “true” regression lines overlaid

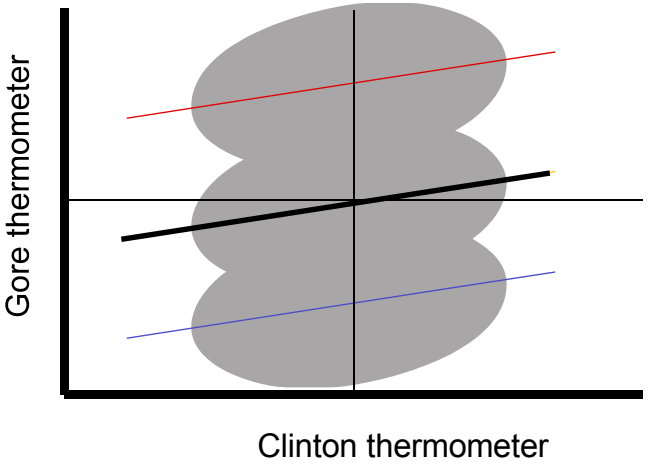


# Tempting yet wrong normalizations

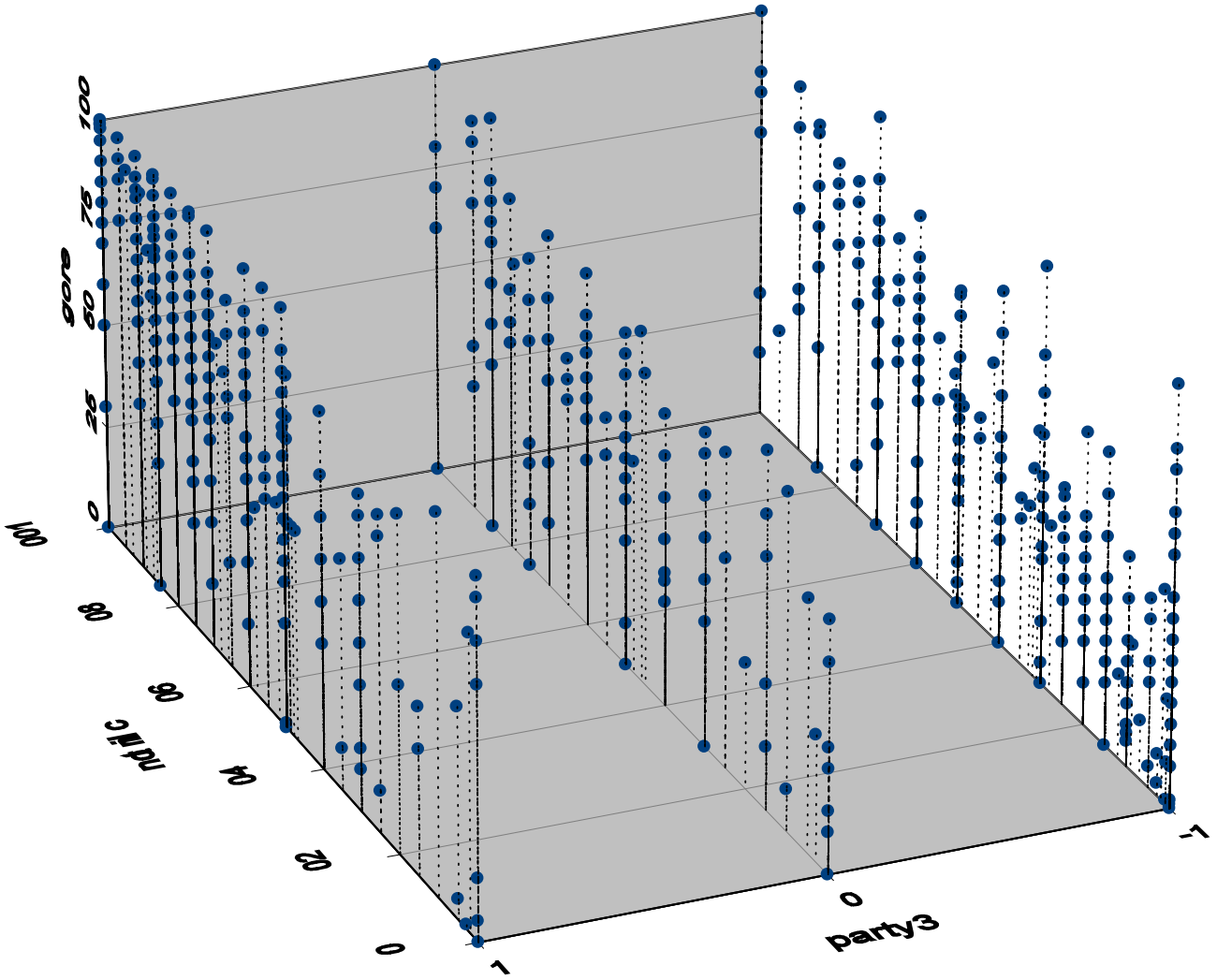
Subtract the Gore therm. from the avg. Gore therm. score




Subtract the Clinton therm. from the avg. Clinton therm. score



# 3D Relationship



# The Linear Relationship between Three Variables

Gore thermometer

Clinton thermometer

Party ID

$$Y_i = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + \varepsilon_i$$

# Multivariate slope coefficients

Clinton effect (on Gore) in bivariate (B) regression

Party ID effect (on Gore) in multivariate (*M*) regression

Bivariate estimate: $\hat{\beta}_1^B = \frac{\text{cov}(X_1, Y)}{\text{var}(X_1)}$ vs.

Multivariate estimate: $\hat{\beta}_1^M = \frac{\text{cov}(X_1, Y)}{\text{var}(X_1)} - \hat{\beta}_2^M \frac{\text{cov}(X_1, X_2)}{\text{var}(X_1)}$

Clinton effect (on Gore) in multivariate (*M*) regression

Clinton effect on Party ID in bivariate regression

When does $\hat{\beta}_1^B = \hat{\beta}_1^M$? Obviously, when $\hat{\beta}_2^M \frac{\text{cov}(X_1, X_2)}{\text{var}(X_1)} = 0$

# The Slope Coefficients

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n}(\bar{Y} - Y_i)(\bar{X}_1 - X_{1,i})}{\sum_{i=1}^{n}(\bar{X}_1 - X_{1,i})^2} - \hat{\beta}_2 \frac{\sum_{i=1}^{n}(\bar{X}_1 - X_{1,i})(\bar{X}_2 - X_{2,i})}{\sum_{i=1}^{n}(\bar{X}_1 - X_{1,i})^2} \text{ and}$$

$$\hat{\beta}_2 = \frac{\sum_{i=1}^{n}(\bar{Y} - Y_i)(\bar{X}_2 - X_{1,i})}{\sum_{i=1}^{n}(\bar{X}_2 - X_{2,i})^2} - \hat{\beta}_1 \frac{\sum_{i=1}^{n}(\bar{X}_1 - X_{1,i})(\bar{X}_2 - X_{2,i})}{\sum_{i=1}^{n}(\bar{X}_2 - X_{2,i})^2}$$

$X_1$ is Clinton thermometer, $X_2$ is PID, and Y is Gore thermometer

# The Slope Coefficients More Simply

$$\hat{\beta}_1 = \frac{\text{cov}(X_1, Y)}{\text{var}(X_1)} - \hat{\beta}_2 \frac{\text{cov}(X_1, X_2)}{\text{var}(X_1)} \text{ and}$$

$$\hat{\beta}_2 = \frac{\text{cov}(X_2, Y)}{\text{var}(X_2)} - \hat{\beta}_1 \frac{\text{cov}(X_1, X_2)}{\text{var}(X_2)}$$

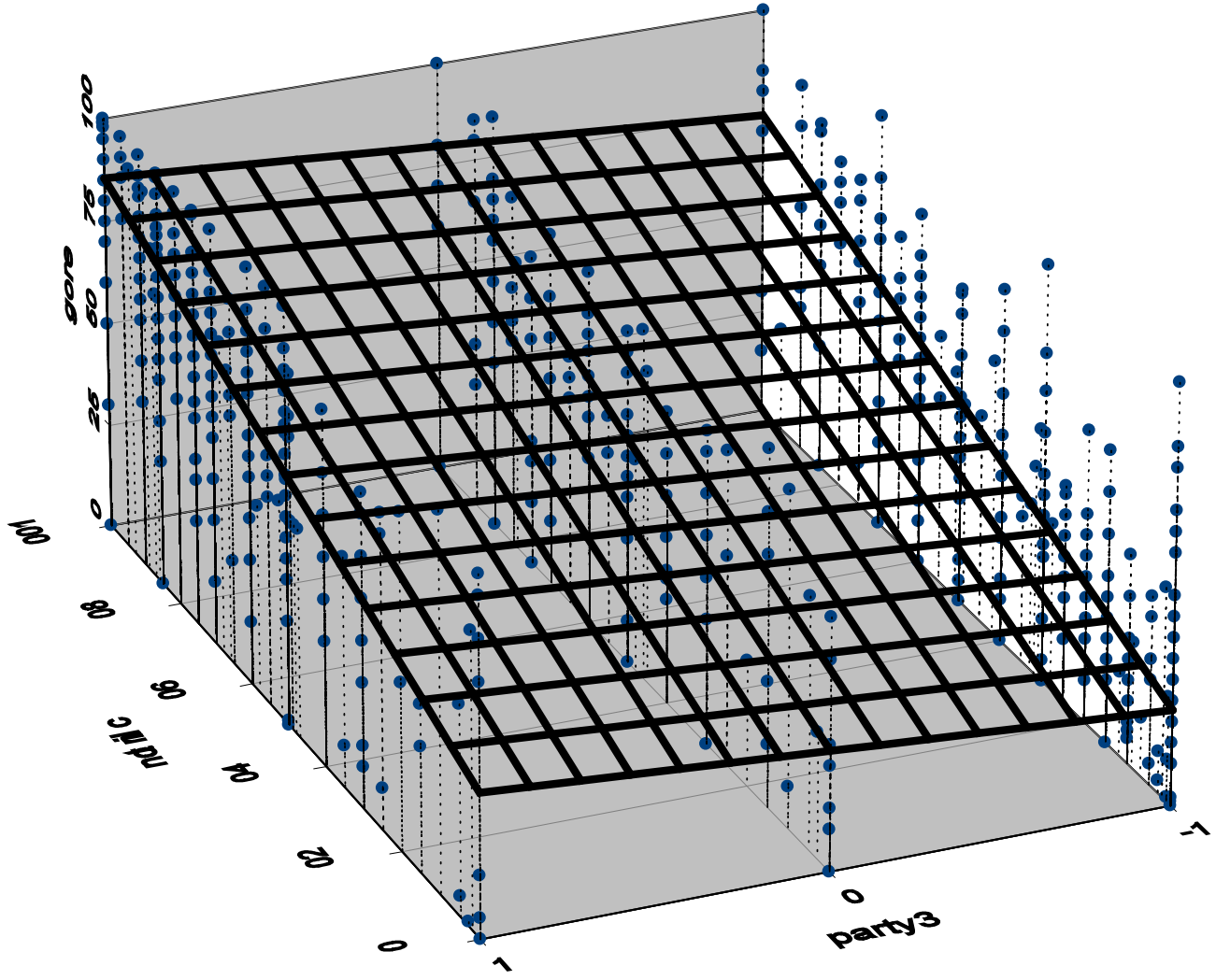
$X_1$ is Clinton thermometer, $X_2$ is PID, and Y is Gore thermometer

# The Matrix form

| | | | | | |
|---|---|---|---|---|---|
| $y_1$ | 1 | $x_{1,1}$ | $x_{2,1}$ | … | $x_{k,1}$ |
| $y_2$ | 1 | $x_{1,2}$ | $x_{2,2}$ | … | $x_{k,2}$ |
| … | 1 | … | … | … | … |
| $y_n$ | 1 | $x_{1,n}$ | $x_{2,n}$ | … | $x_{k,n}$ |

$$\beta = (X'X)^{-1} X'y$$

# 3D Linear Relationship



# The Output

```
. reg gore clinton party3

      Source |       SS       df       MS              Number of obs =    1745
-------------+------------------------------           F(  2,  1742) = 1048.04
       Model |   629261.91     2  314630.955           Prob > F      =  0.0000
    Residual |  522964.934  1742  300.209492           R-squared     =  0.5461
-------------+------------------------------           Adj R-squared =  0.5456
       Total |  1152226.84  1744   660.68053           Root MSE      =  17.327

------------------------------------------------------------------------------
        gore |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     clinton |   .5122875   .0175952    29.12   0.000     .4777776    .5467975
      party3 |   5.770523   .5594846    10.31   0.000     4.673191    6.867856
       _cons |    28.6299   1.025472    27.92   0.000     26.61862    30.64119
------------------------------------------------------------------------------
```

Interpretation of `clinton` effect: *Holding constant party identification*, a one-point increase in the Clinton feeling thermometer is associated with a .51 increase in the Gore thermometer.

# Separate regressions

| | (1) | (2) | (3) |
|---|---|---|---|
| Intercept | 23.1 | 55.9 | 28.6 |
| Clinton | 0.62 | -- | 0.51 |
| Party | -- | 15.7 | 5.8 |

# Is the Clinton effect causal?

- That is, should we be convinced that negative feelings about Clinton really hurt Gore?
- No!
  - The regression analysis has only ruled out nonrandom selection on party ID.
  - Nonrandom selection into the treatment could occur from
    - Variables other than party ID, or
    - Reverse causation, which is feelings about Gore influencing feelings about Clinton.
  - Additionally, the regression analysis may not have entirely ruled out nonrandom selection on party ID because it may have assumed he wrong functional form.
    - E.g., what if nonrandom selection on strong Republican/strong Democrat

# Summary:  Why we control

- Address alternative explanations by removing confounding effects
- Improve efficiency

# Why did the Clinton Coefficient change from 0.62 to 0.51

```
. corr gore clinton party, cov
(obs=1745)

             |     gore  clinton   party3
-------------+---------------------------
        gore |  660.681
     clinton |  549.993  883.182
      party3 |  13.7008   16.905    .8735
```

# The Calculations

$$\hat{\beta}_1^B = \frac{\text{cov}(gore, clinton)}{\text{var}(clinton)} = \frac{549.993}{883.182} = 0.6227$$

$$\hat{\beta}_1^M = \frac{\text{cov}(gore, clinton)}{\text{var}(clinton)} - \hat{\beta}_2^M \frac{\text{cov}(clinton, party)}{\text{var}(clinton)}$$

$$= \frac{549.993}{883.182} - 5.7705 \frac{16.905}{883.182}$$

$$= 0.6227 - 0.1105$$

$$= 0.5122$$

```
. corr gore clinton party,cov
(obs=1745)

             |     gore  clinton   party3
-------------+---------------------------
        gore |  660.681
     clinton |  549.993  883.182
      party3 |  13.7008   16.905    .8735
```

# Accounting for total effects

$$\hat{\beta}_1^M = \frac{\mathrm{cov}(X_1, Y)}{\mathrm{var}(X_1)} - \hat{\beta}_2 \frac{\mathrm{cov}(X_1, X_2)}{\mathrm{var}(X_1)}$$

(i.e., regression coefficient when we regress $X_2$ (as dep. var.) on $X_1$ (as ind. var.)

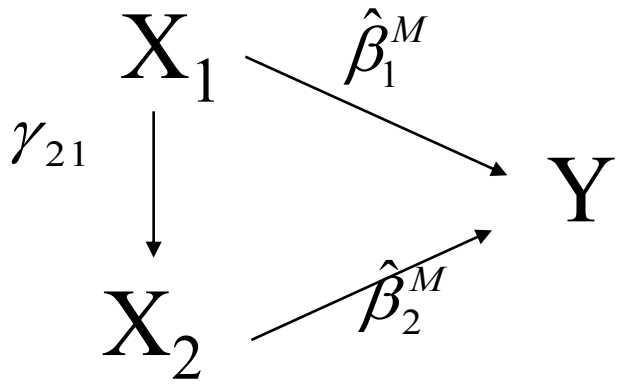
$$\hat{\beta}_1^M = \hat{\beta}_1^B - \hat{\beta}_2^M \gamma_{21}^M$$

$$\hat{\beta}_1^B = \hat{\beta}_1^M + \hat{\beta}_2^M \gamma_{21}^M$$

# Accounting for the total effect

$$\hat{\beta}_1^B = \hat{\beta}_1^M + \hat{\beta}_2^M \gamma_{21}$$

Total effect = Direct effect + indirect effect



# Accounting for the total effects in the Gore thermometer example

| Effect | Total | Direct | Indirect |
|---|---|---|---|
| Clinton | 0.62 | 0.51 | 0.11 |
| Party | 15.7 | 5.8 | 9.9 |

# Other approaches to addressing confounding effects?

- Experiments
- Difference-in-differences designs
- Others?

- Is regression the best approach to addressing confounding effects?
  - Problems

# Drinking and Greek Life Example

- Why is there a correlation between living in a fraternity/sorority house and drinking?
  - Greek organizations often emphasize social gatherings that have alcohol. The effect is being in the Greek organization itself, not the house.
  - There's something about the House environment itself.

# Dependent variable: Times Drinking in Past 30 Days

**C8. When did you last have a drink (that is more than just a few sips)?**

- ◯ I have never had a drink → Skip to **C22** (page 10)
- ◯ Not in the past year → Skip to **C22** (page 10)
- ◯ More than 30 days ago, but in the past year → Skip to **C17** (page 8)
- ◯ More than a week ago, but in the past 30 days → Go to **C9**
- ◯ Within the last week → Go to **C9**

**C9. On how many occasions have you had a drink of alcohol in the <u>past 30 days</u>? (Choose one answer.)**

- ◯ Did not drink in the last 30 days
- ◯ 1 to 2 occasions
- ◯ 3 to 5 occasions
- ◯ 6 to 9 occasions
- ◯ 10 to 19 occasions
- ◯ 20 to 39 occasions
- ◯ 40 or more occasions

```
. infix age 10-11 residence 16 greek 24 screen 102
timespast30 103 howmuchpast30 104 gpa 278-279 studying 281
timeshs 325 howmuchhs 326 socializing 283 stwgt_99 475-493
weight99 494-512 using da3818.dat,clear
(14138 observations read)

. recode  timespast30 timeshs (1=0) (2=1.5) (3=4) (4=7.5)
(5=14.5) (6=29.5) (7=45)
(timespast30: 6571 changes made)
(timeshs: 10272 changes made)

. replace timespast30=0 if screen<=3
(4631 real changes made)
```

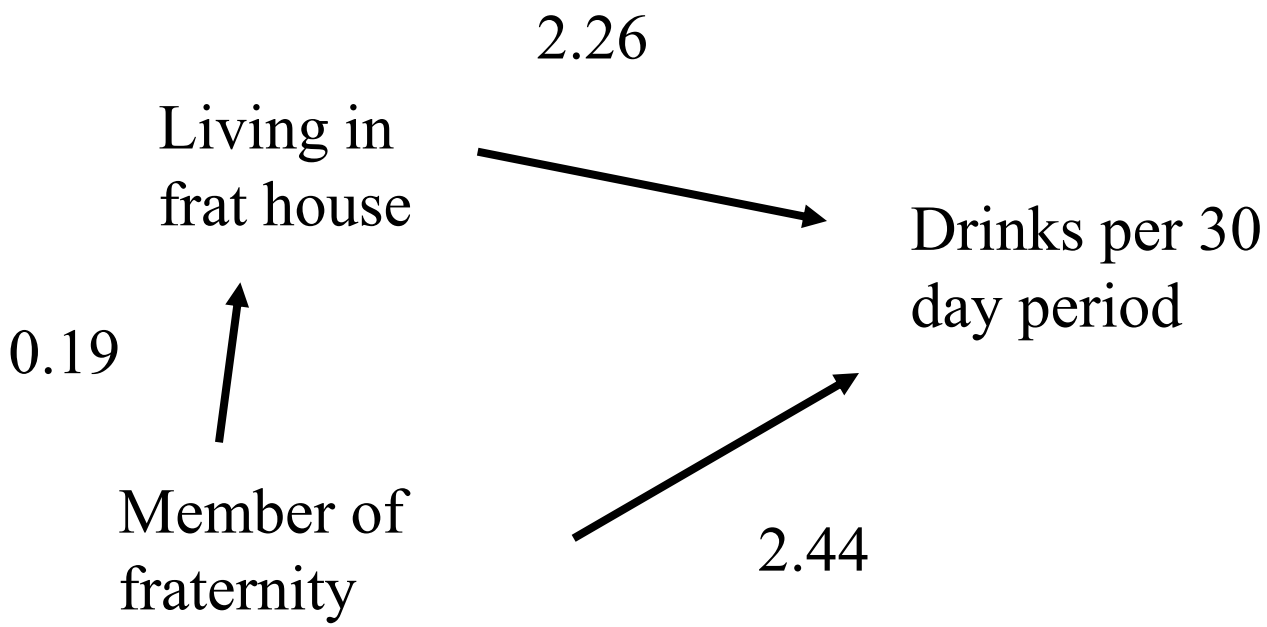
```
. tab timespast30

timespast30 |      Freq.     Percent        Cum.
------------+-----------------------------------
          0 |      4,652       33.37       33.37
        1.5 |      2,737       19.64       53.01
          4 |      2,653       19.03       72.04
        7.5 |      1,854       13.30       85.34
       14.5 |      1,648       11.82       97.17
       29.5 |        350        2.51       99.68
         45 |         45        0.32      100.00
------------+-----------------------------------
      Total |     13,939      100.00
```

# Three Regressions

| Dependent variable: number of times drinking in past 30 days | | | |
|---|---|---|---|
| Live in frat/sor house | 4.44<br>(0.35) | --- | 2.26<br>(0.38) |
| Member of frat/sor | --- | 2.88<br>(0.16) | 2.44<br>(0.18) |
| Intercept | 4.54<br>(0.56) | 4.27<br>(0.059) | 4.27<br>(0.059) |
| R2 | .011 | .023 | .025 |
| N | 13,876 | 13,876 | 13,876 |

Note: Corr. Between living in frat/sor house and being a member of a Greek organization is .42

# The Picture


2.26
Living in frat house
Drinks per 30 day period
0.19
Member of fraternity
2.44

# Accounting for the effects of frat house living and Greek membership on drinking

| Effect | Total | Direct | Indirect |
|---|---|---|---|
| Member of Greek org. | 2.88 | 2.44 (85%) | 0.44 (15%) |
| Live in frat/ sor. house | 4.44 | 2.26 (51%) | 2.18 (49%) |